



## Socratic Mathematics with Bill Carey

### Session 1: Figurate Numbers

#### Outline:

##### Figurate Numbers

- For this first session, we'll begin with what the Greeks called figurate numbers. Here are the first four **triangular numbers**:



- You'll want to copy those onto your whiteboard so that everyone in the group can see them clearly. They're the mathematical objects you'll be discussing during this session.
- **Contemplation:** Invite the group to spend a few minutes – four or five is usually fruitful – contemplating the first four triangular numbers. Ask folks to look for patterns and structure, and to jot down things they notice. After the five minutes, folks should share what they see with the group.
- **Discussion Questions:** Remember that the goal here is to seek out truth together, and convince yourselves that you've found it. As the facilitator, part of your responsibility is to make sure that everyone in the group is heard and on board!
  - What is the next (i.e. the fifth) triangular number?
  - What is the 113<sup>th</sup> triangular number? There's a slow and laborious way to find this. Can you find an easier way?
  - What is the  $n^{\text{th}}$  triangular number? This is a generalization of the previous question. Can you figure out a quick way to find *any* triangular number at all?
- **Conclusion:** After you've come to a good consensus about the third question, share this quotation from Nicomachus's *Arithmetic* with the group:



Every square figure <sup>1</sup> diagonally divided is resolved into two triangles and every square number is resolved into two consecutive triangular numbers, and hence is made up of two successive triangular numbers. For example, 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, and so on, are triangular numbers and 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, squares. If you add any two consecutive triangles that you please, you will always make a square, and hence, whatever square you resolve, you will be able to make two triangles of it.

- What does Nicomachus mean here? Do you all think about triangular numbers in the same way that Nicomachus does? What do you all make of that?