Outline:

Square Numbers

- For this next session, we'll look at the square numbers. (It blew my mind when I found out that three squared was an actual square.) Here are the first four **square numbers**:
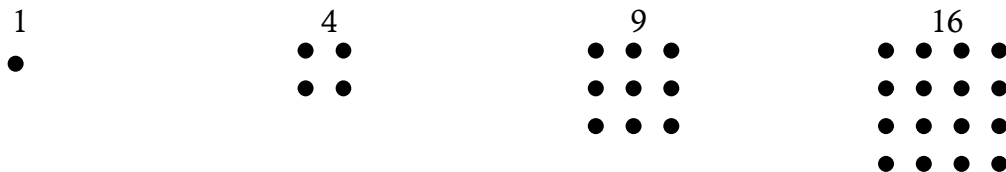
- You'll want to copy those onto your whiteboard so that everyone in the group can see them clearly. They're the mathematical objects you'll be discussing during this session.
- **Contemplation:** Invite the group to spend a few minutes – two or three is usually fruitful – contemplating the first four square numbers. In some ways they're simpler than the triangular numbers! Ask folks to look for patterns and structure, and to jot down things they notice. After the five minutes, folks should share what they see with the group.
- **Discussion Questions:** Remember that the goal here is to seek out truth together, and convince yourselves that you've found it. As the facilitator, part of your responsibility is to make sure that everyone in the group is heard and on board!
  - What is the next (i.e. the fifth) square number?
  - Why is it easier to find the 113th square number than the 113th triangular number? How would you find the 113th square number?
  - What is the nth square number? Easier or harder than for triangular numbers?
  - Let's say that I tell you the 202nd square number. It's 40,804. Could you find out the 203rd square number in an easy way? Maybe even in your head? (This is also secretly a question about generalization: how do you go from the nth square number to the $(n+1)^{th}$? Why does that work?
- **Conclusion:** After you've come to a good consensus about the third question, share this quotation from Nicomachus's *Arithmetic* with the group:

This number also is produced[1] if the natural series is extended in a line, increasing by 1, and no longer the successive numbers are added to the numbers in order, as was shown before, but rather all those in alternate places, that is, the odd numbers. For the first, 1, is potentially the first square; the second, 1 plus 3, is the first in actuality; the third, 1 plus 3 plus 5, is the second in actuality; the fourth, 1 plus 3 plus 5 plus 7, is the third in actuality; the next is produced by adding 9 to the former numbers, the next by the addition of 11, and so on.

- What does Nicomachus mean here? Do you all think about square numbers in the same way that Nicomachus does? What do you all make of that?