Outline:

Co-primality

- In this third session, we're going to explore a **definition**, and see how it plays out in particular examples. Mathematicians like to move between the general and the particular, so we'll build up some of those muscles thinking about **co-primality**.
- For the contemplation part of the discussion, we'll start with the following particular examples. We don't know a rule yet (or even what co-primality means). But we can craft some conjectures together!

Here are some pairs of numbers that **are** co-prime:

4 and 3	6 and 5	11 and 3	17 and 14
5 and 2	7 and 4	17 and 2	21 and 20

Here are some pairs of numbers that **are not** co-prime:

4 and 2	6 and 3	12 and 8	21 and 14
6 and 2	9 and 6	21 and 3	52 and 13

- **Contemplation:** Invite the group to spend a few minutes – five to eight is usually fruitful – contemplating these groups of numbers and looking for patterns and structure. Is there a rule that governs whether or not two numbers are co-prime?
- **First Discussion:** Collectively try to write out a rule that sorts pairs of numbers into coprime and not co-prime. Don't feel obliged to use "mathematical" language: your goal is clarity and understanding. Conjecture boldly! Refine them in charity. Once you've come to consensus about your rule, you can play with it!
  - Now that you have a conjectured rule everyone is happy with, try to break it! Propose pairs of numbers and see whether they fit the definition well.
  - Can you make your rule shorter while saying the same thing?

- How does your rule categorize 0.4 and 1.2?
- **Second Discussion:** It's always fun to compare our conjectures and ideas with mathematicians of yore. We participate in a great conversation across the millennia. You can play with Euclid's definition of co-primality:

Euclid VII, Def. 12: Numbers co-prime are those which are measured by a unit alone as a common measure.

- Does this say the same thing as your rule? Does it say it better? Worse? What are the trade-offs? What words in Euclid's definition would you want to understand more?
- **Conclusion:** Having a general rule is fantastic. The best way to cement our shared understanding of a general rule is to try and apply it to particulars. Here are a few you can play with!
  - How many numbers less than 21 are co-prime with 21?
  - How many numbers less than 22 are co-prime with 22?
  - How many numbers less than 23 are co-prime with 23?
  - How many numbers less than 24 are co-prime with 24?
  - Wait, what?
- Are 11,746 and 9,873 co-prime? How would you decide? Is there an easier way? (There is, and Euclid found it!)