Socratic Mathematics
with Bill Carey

Session 5: A Numerical Puzzle

Outline:

- **Contemplation:** For this session, I'm going to offer up two puzzles that you can read aloud to the group, or perhaps write out on your shared whiteboard. The goal of contemplating these is *not* to solve them, but to look at the features of the puzzle itself. How does it land on your ears? What does it make you feel? What jumps out at you? What is confusing?
- "If I tell you that I'm thinking of two numbers, and that they add up to 100 and that the difference between them is 40, could you tell me the numbers?"
- "Two neighboring farmers wish to purchase a draft horse to share from their duke for the price of 100 bezants. The richer of the two farmers will pay the ducal taxes, thus paying 40 more bezants than the poorer farmer. What does each man pay the duke for the horse?"
- **Discussion Questions:** Remember that the goal here is to seek out truth together, and convince yourselves that you've found it. As the facilitator, part of your responsibility is to make sure that everyone in the group is heard and on board!
    - If I tell you that I'm thinking of two numbers, and that they add up to 100 and that the difference between them is 40, could you tell me the numbers?
    - If I tell you that I'm thinking of two numbers, and that they add up to 18 and that the difference between them is 4, could you tell me the numbers?
    - If I tell you that I'm thinking of two numbers, and that they add up to 17 and that the difference between them is 4, could you tell me the numbers?
    - If I tell you that I'm thinking of two numbers, and that they add up to 12 and that the difference between them is 3, could you tell me the numbers?
    - Can you craft some conjectures about the difference between the first two questions and the second two questions? Feel free to play around with more examples of your own looking for patterns! (I don't want to be too specific here, as there are a couple of interesting conjectures!)
    - Can you describe a general rule that you could use to solve a puzzle like this whatever the numbers add up to and whatever their difference is?
- **Conclusion:** This is the first puzzle in Diophantus's book of Arithmetic. There are many more like it! Here's how he tackles this one:

To separate a given number into two numbers having a given difference.

- **Diophantus's Argument**

Given number: 100, given difference 40.
[I] Let the lesser number required be x.

[II] The greater number is then $x + 40$.

[III] Then $2x + 40 = 100$.

[IV] Therefore $x = 30$.

Thus the required numbers are 30 and 70.

Did you think about it in the same way? Do you find the language and symbols Diophantus uses helpful or confusing?