Socratic Mathematics

with Bill Carey

Session 6: Another Numerical Puzzle

Outline:

Another Numerical Puzzle

- **Contemplation:** For this session we're going to look at *proportions*. Here are a few that you can copy onto your shared whiteboard for everyone to contemplate:

$$\frac{1}{2} = \frac{2}{4} \qquad \frac{4}{6} = \frac{6}{9} \qquad \frac{7}{21} = \frac{21}{63}$$

$$\frac{1}{7} = \frac{7}{49} \qquad \frac{5}{15} = \frac{15}{14}$$

- They're worth a few minutes of contemplation. Hopefully, by this session, you've grown into a good pattern of sharing observations, patterns, questions, and the like. The goal of this time is to bring ourselves to the mathematical objects without any particular question and see how they work on us, just like you would with a poem.
- **Discussion Questions:** Remember that the goal here is to seek out truth together, and convince yourselves that you've found it. As the facilitator, part of your responsibility is to make sure that everyone in the group is heard and on board!
    - Can you make up a few more proportions that follow the rule the examples do?
    - Could you make up a proportion with *letters* that makes that rule general?
    - Can you craft a conjecture that relates those proportions to the square numbers we talked about in the second session?
    - The proportion $\frac{4}{6} = \frac{6}{9}$ is interesting. I could write a different proportion that has six in the same places: $\frac{2}{6} = \frac{6}{18}$. But for 7, I can't do that. $\frac{1}{7} = \frac{7}{49}$ is the only proportion with seven in those places. What's up with that?

- **Conclusion:** The general form of these proportions is $\frac{a}{b} = \frac{b}{c}$, which we could also write like this: $ac = b^2$.