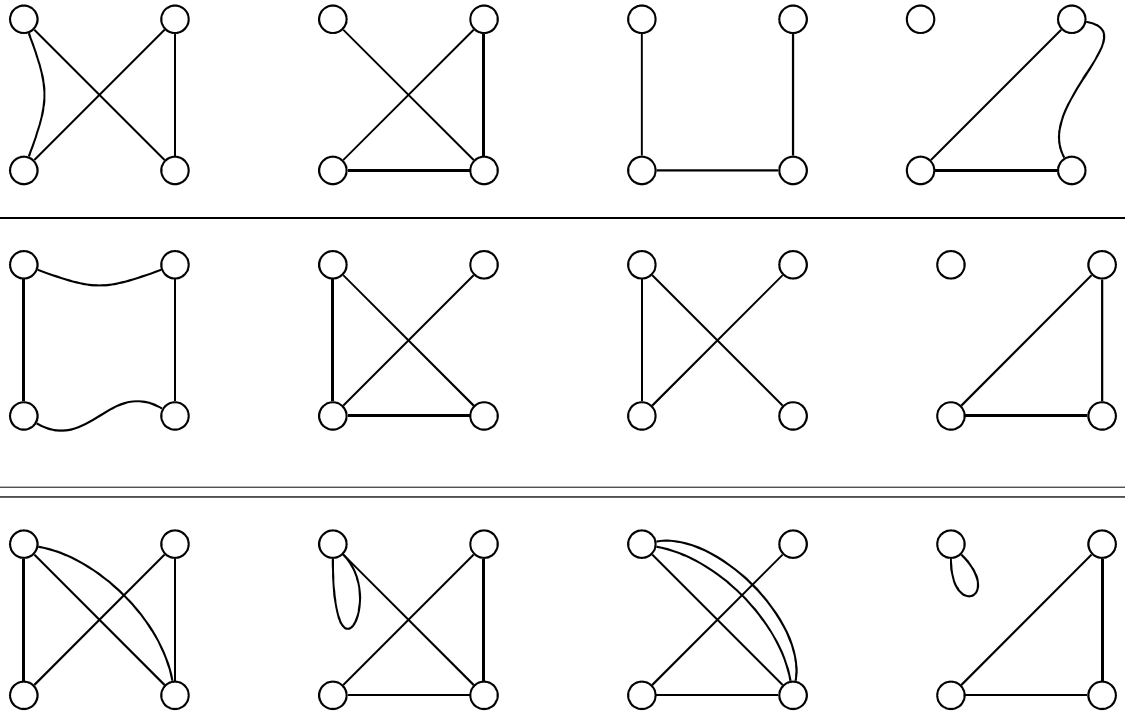
Socratic Mathematics
with Bill Carey

Session 8: Patterns in Graphs

Outline:

Patterns in Graphs

- **Contemplation:** In the last session we looked at **graphs**. We're going to continue to play with them in this session! The mathematical objects to contemplate this session are the graphs below. Each graph has four dots and some lines. Here's what you'll want to share with your fellow mathematicians:



- It's worth a few minutes of contemplation. Hopefully, by this session, you've grown into a good pattern of sharing observations, patterns, questions, and the like. The goal of this time is to bring ourselves to the mathematical objects without any particular question and see how they work on us, just like you would with a poem.
- **First Discussion Questions:** Remember that the goal here is to seek out truth together, and convince yourselves that you've found it. As the facilitator, part of your responsibility is to make sure that everyone in the group is heard and on board!

  - Can you order the graphs from fewest lines to most lines? Can there be more than one graph with the same number of lines?
  - In the top two rows, the graphs above and below each other are the *same* graph. So the top left-most and the middle left-most are the same graph. Given that, can you conjecture what counts when distinguishing graphs? Could you draw any of those graphs in different ways?
- **Second Discussion Questions:** If you're happy with the discussion about when graphs are the same or different, here are a few more that you can play with to tease out some more properties of graphs. Let's see if we can find some rules to categorize them!
  - The graphs on the top are all happy graphs, and the graphs on the bottom are all sad graphs. Can you come up with a rule to sort graphs into happy and sad?
  - There are exactly twelve different happy graphs that have four dots. I've drawn four of them for you. Can you find the other eight?
  - How many happy graphs are there with exactly three dots? How many with exactly two? How many with exactly one?
- **Conclusion:** Euler's Graph theory has been one of the most fruitful developments in the last several hundred years of mathematics. It's a direct descendant of the Greek conception of the discrete in arithmetic. (Why?) Think about how many things in your life could be represented as graphs! What could the dots represent? What the about lines?