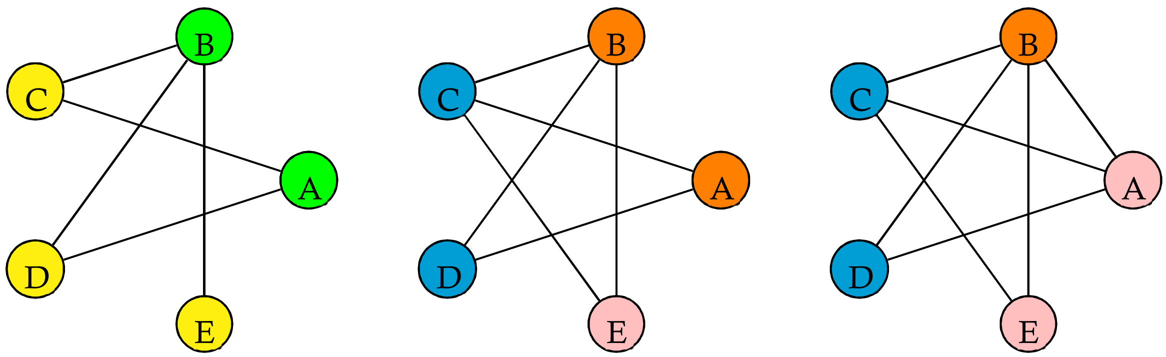
Socratic Mathematics
with Bill Carey

Session 9: Chromatic Graphs

Outline:

Chromatic Graphs

- For this last session we'll continue to play around with graphs. Here are the graphs we'll noodle through during this session:



- **Contemplation:** These graphs repay some contemplation well. There is much to notice. Hopefully, by this session, you've grown into a good pattern of sharing observations, patterns, questions, and the like. The goal of this time is to bring ourselves to the mathematical objects without any particular question and see how they work on us, just like you would with a poem or painting.
- After you've spent some time noodling through those graphs, share this definition with the group:

Chromatic Number: The chromatic number of a graph is the smallest number of colors you'd need to color in each dot so that no two connected dots are the same color.

- **First Discussion Questions:** As always, the goal here is to seek out truth together, and convince yourselves that you've found it. As the facilitator, part of your responsibility is to make sure that everyone in the group is heard and on board!
  - What's the chromatic number of each of those graphs?

  - Could you sketch a graph whose chromatic number is one?
  - What's the smallest, simplest graph whose chromatic number is three?
  - Imagine you've got a graph that represents the corners of a cube connected by the edges of the cube (you could grab a box or some dice or something to be your cube). What's the chromatic number of that graph?
  - Could you color in any of the graphs you've played with in different ways that still use the same number of colors?
  - Attached is a map of the United States. How many different colors would you need to color in each state a different color from its neighbors?
- **Conclusion:** The map coloring problem originated in England in 1852. A student of Augustus DeMorgan (famous for his logical laws) asked DeMorgan whether four colors could color any map. DeMorgan wasn't sure, so asked his colleagues. In 1878 (!) a mathematician named Kempe published a paper claiming to prove that four colors were always enough. His argument stood until 1890 (!) when Percy Harwood noted a problem with his argument. The question stood unresolved until 1976 (!!) when Appel and Haken put forward a paper based on a computer program purporting to prove that four colors were enough. Whether that proof works is still debated, though improved versions of their argument tilt the scales towards "yes". Some good math problems take *time*, which is a lovely thing.

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