CLASSICALU

Mathematics for Every Teacher

with Jake Tawney

Lecture 11: Fermat's Last Theorem

Outline:

Fermat's Last Theorem, The Pythagorean Theorem Up a Notch

- **The Pythagorean Theorem**: In a right triangle, the sum of the square on one leg and the square on the other leg is equal to the square on the hypotenuse.
    - $a^2 + b^2 = c^2$
    - $3^2 + 4^2 = 5^2$ → Pythagorean Triples
    - $3n^2 + 4n^2 = 5n^2$
- **Primitive Pythagorean Triples**
    - $a = m^2 - n^2$
    - $b = 2\ x\ m\ x\ n$
    - $c = m^2 + n^2$
    - $m > n$
    - Choose m and n so that (1) they have no factors in common, and (2) they are both not odd.
    - $a^2 + b^2 = c^2$ → There are an infinite number of triples and primitive Pythagorean triples (for positive integer solutions).
- $a^3 + b^3 = c^3$ → Try as they might, mathematicians did not have a positive integer solution.
    - $a^4 + b^4 = c^4$ → No positive integer solutions, but mathematicians could not find a proof for why not.
    - "It is not possible to separate a cube into two cubes, a fourth power into two fourth powers, or in general any power greater than 2 into two like powers."
    - Fermat writes that he has a "remarkable proof" (*demonstationem mirabilem*) of this, but "*Hanc marginis exiguitas non caperet*," meaning, "This smallness of this margin cannot contain it."
    - **Fermat's Last Theorem**: If n is greater than 2, then there are no positive integer solutions for a, b, and c in the equation $a^n + b^n = c^n$.