Mathematics for Every Teacher

with Jake Tawney

Lecture 13a: A Handful of Unsolved Problems: The Infinite Depth of Mathematical Mystery

Outline:

A Handful of Unsolved Problems

- First Unsolved Problem:
  - How can I figure out the shortest distance from the lower corner on one side to the upper corner on the other side of the box when the side lengths are known?
  - **The 3-D Pythagorean Theorem**: If a, b, and c, are the three dimensions of a box and d is the diagonal from one corner of the box to its opposite corner, then $d^2 = a^2 + b^2 + c^2$.
  - **Definition**: A *Pythagorean quadruple* is a group of four whole numbers a, b, c, and d such that $a^2 + b^2 + c^2 = d^2$.
  - The other face diagonals are not whole numbers. They are square roots that are not rational.
  - **Definition**: An *Euler brick* is a three-dimensional box that has all whole number sides and all whole number face diagonals.
  - **Definition**: A *perfect Euler brick* is a rectangular box that has all whole number sides, all whole number face diagonals, and a whole number main diagonal.
  - **Unsolved Problem #1**: Does a perfect Euler brick exist?
- Second and Third Unsolved Problems:
  - There are an infinite number of prime numbers.
  - The Prime Number Theorem allows us to measure the average gap between primes below a certain number.
  - After six trillion we find two primes that are mysteriously close together.
  - **Bertrand-Chebyshev Theorem**: Given any n greater than 3, there is always a prime between n and $2n - 2$.
  - **A Theorem About Prime Gaps**: We can find arbitrarily large gaps in the prime numbers.
  - **Bonus Unsolved Problem: Legendre's Conjecture**: There is always a prime number between n^2 and $(n + 1)^2$ for every whole number n.
  - **Definition**: Two primes are called *twin primes* if they are exactly two apart.
  - **Unsolved Problem #2**: Are there a finite or an infinite number of twin prime pairs?

- **Unsolved Problem #3 (Goldbach Conjecture)**: Any even integer greater than 2 can be written as the sum of two prime numbers.
  - The Goldbach Conjecture seems to be ridiculously true. This could be part of why it is so difficult to prove this.