Mathematics for Every Teacher

with Jake Tawney

Lecture 13b: A Handful of Unsolved Problems: The Infinite Depth of Mathematical Mystery

Outline:

Continuation of a Handful of Unsolved Problems

- Fourth Unsolved problem
  - Definition: *Proper divisors* are all the divisors for a given number excluding the number itself.
  - Definition: A *perfect number* is when the sum of the proper divisors equals the original number. For example, the divisors of 6 are 1,2 and 3. 1+2+3=6 so 6 is a perfect number.
  - Definition: An *abundant number* is when the sum of the proper divisors is greater than the original number.
  - Definition: A *deficient number* is when the sum of the proper divisors is less than then original number.
  - Euclid defines a perfect number as a number hat is the sum of its parts.
  - Euclid states if the sum of the double is prime multiplying the sum by

Doubles	Sum of Doubles	Is the sum prime?	Sum × Last Double (perfect number)
1, 2	1 + 2 = 3	3 is prime	3 × 2 = 6
1, 2, 4	1 + 2 + 4 = 7	7 is prime	7 × 4 = 28
1, 2, 4, 8	1 + 2 + 4 + 8 = 15	15 is NOT prime	N/A
1, 2, 4, 8, 16	1 + 2 + 4 + 8 + 16 = 31	31 is prime	31 × 16 = 496
1, 2, 4, 8, 16, 32	1 + 2 + 4 + 8 + 16 + 32 = 63	63 is NOT prime	N/A
1, 2, 4, 8, 16, 32, 64	1 + 2 + 4 + 8 + 16 + 32 + 64 = 127	127 is prime	127 × 64 = 8,128

The Euclid-Euler Theorem
When consecutive powers of 2 are added together to get a prime number, the prime number multiplied by the last power of 2 will always be an even perfect number. Moreover, this process will produce *all the even perfect numbers*.

the last of the doubles will give you a prime number.

- **Unsolved problem #4a**: Are there an infinite number of even perfect numbers?
    - Does this process eventually not produce a prime number?
- **Bonus question**: Are there an infinite number of Mersenne Primes?
- **Unsolved problem #4b**: Are there any odd perfect numbers?
    - We do not know if any exist nor have, we been able to prove they do not exist.

- Fifth Unsolved problem
    - **Collatz Conjecture**
        - Start with any positive integer
        - If it is even, divide the number by 2
        - If it is odd, multiply it by 3 and add 1
        - Repeat process with new number
        - Eventually the repeated process will always end in 1
    - Definition: *Hailstone numbers* are an analogy describing a sequence of numbers where the evens go up and the odds go down, eventually gravity takes over and the numbers fall to 1
    - **Unsolved Problem #5**: Is Collatz Conjecture always true?
        - Do the numbers eventually get so high that the pull up is stronger than the pull down?
        - Do some numbers eventually for a loop that does not go to 1?
    - Modifying the conjecture from 3n+1 to 3n-1 causes a loop.
    - Modifying the conjecture from 3n+1 to 6n+1 goes to infinity
    - Why do we work on Collatz Conjecture?
        - Solving this could give us deeper insight into how easy it is to avoid powers of 2.
        - The deep and wonderful mathematics that is needed to solve it.
        - Mere curiosity, it is an easy problem but difficult to solve.
    - It is intriguing to younger students because you only need arithmetic to work with it to try and solve it.