



# Mathematics for Every Teacher

with Jake Tawney

## Lecture 7: The Independence of the Parallel Postulate

### Outline:

The Independence of the Parallel Postulate, How to Prove Something Cannot be Proven

- When Euclid set out to create his masterpiece, he looked at first for starting places: definitions, common notions, and postulates.
  - 5 Postulates as Euclid's Starting Place:
    - To draw a straight line from any point to any point.
    - To produce a finite straight line continuously in a straight line.
    - To describe a circle with any center and distance.
    - That all right angles are equal to one another.
    - **The Parallel Postulate:** That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.
- Mathematicians set out to prove Euclid's fifth postulate.
  - **Playfair's Axiom:** In a plane, given a line and point not on the line, there is at most one line through the point parallel to the given line.
  - **Alternate Formulation:** In a plane, given a line and point not on the line, there is *exactly one* line through the point parallel to the given line.
- The first four postulates could not be used to prove that the Parallel Postulate is true, nor could they be used to prove that the Parallel Postulate is false (Beltrami).
  - The Parallel Postulate is independent of the other four.
  - The Parallel Postulate does not hold on the surface of sphere.
  - Beltrami's model is infinite space (hyperbolic space).
    - In Beltrami's model, any triangle has angles that add to less than  $180^\circ$ .
    - In hyperbolic geometry, any regular polygon can be made to tile the hyperbolic plane.
  - **The Independence of the Parallel Postulate:** Euclid's first four postulates cannot be used to prove that the fifth postulate is true, neither can they be used to prove that the fifth postulate is false.
    - The Parallel Postulate is independent of the other four.
- Mathematics can be used to prove that some things are neither provable or disprovable.