## Outline:

Impossible Constructions, How Modern Mathematics Solved an Ancient Problem

- Euclid's first three postulates:
  - To draw a straight line from any point to any point.
  - To produce a finite straight line continuously in a straight line.
  - To describe a circle with any center and distance.
- The first two postulates says that lines can be drawn.
- The third postulate says that circles can be drawn.
- Euclid's First Proposition:
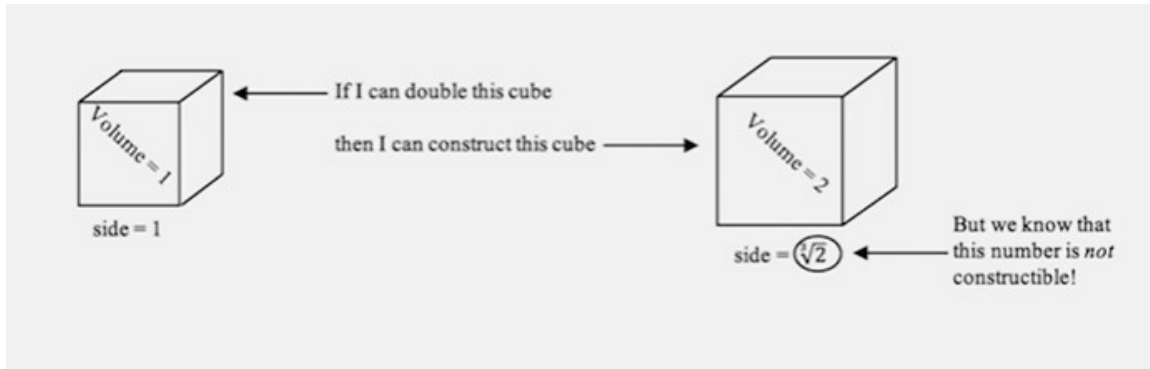 - Euclid makes an equilateral triangle using radii of circles drawn around a line segment.

Three constructions that eluded Euclid:

- Given any angle, divide it equally into three.
- Given a circle, draw a square that has the same area.
- Given a cube, draw another cube that has twice the volume.
- Fermat Primes: A Fermat prime is a prime number that is in the form:
  - $2^{2^n} + 1$
  - When they are prime, they count as Fermat Primes.
- **Gauss' Theorem About Constructible Regular Polygons**: A regular n-gon is constructible if n is the product of any number of 2s as well as distinct Fermat primes.
  - The Greek's knew about at least part of this theorem. They did not know as many Fermat primes.
- **Gauss-Wantzel Theorem**: If n is the product of any number of 2s together with distinct Fermat primes, then a regular n-gon is constructible. Moreover, these are the only constructible regular polygons.
- **How is possible to prove that something is impossible**?
  - Definition: A number is a constructible number if a segment of that length can be drawn using only a straight edge and a compass.
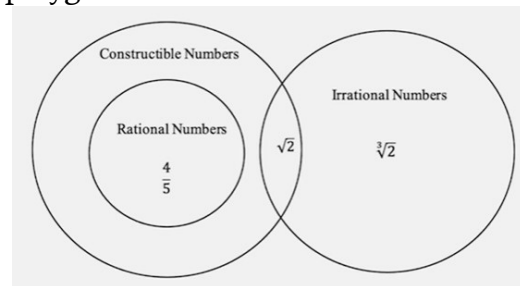
- Addition, subtraction, multiplication, division, and square roots can be constructed with the straight edge and a compass.

If I can double this cube

then I can construct this cube

Volume = 1
side = 1

Volume = 2
side = $\sqrt[3]{2}$

But we know that this number is *not* constructible!

- If doubling the cube is possible then the cube root of two is a constructible number, but the cube root of 2 is not a constructible number.
- Geometry and number theory are joined here by Gauss.
- If you change your tools, you can trisect your angle. If you replace the circle with a folding tool, you can trisect an angle. Yet with those tools, other constructions become impossible.