# Mathematics for Every Teacher

with Jake Tawney

## Lecture 9: Three Fundamental Theorems

### Outline:

Three Fundamental Theorems

- The Fundamental Theorem of Algebra
  - "nice" = polynomials
  - Do all polynomials built with integers have a "nice" solution?
  - $x^2 = -1 \rightarrow x = i$
  - **The Not-Quite-Fundamental Theorem of Algebra**: Any polynomial equation put together with real numbers will have at least one solution in the complex numbers.
  - **The Fundamental Theorem of Algebra**: Any polynomial equation put together with complex numbers will have at least on solution in the complex numbers.
- The Fundamental Theorem of Calculus
  - What is the slope of the tangent line?
  - What is the area under a curve?
  - These two problems are opposite of each other.
- The Fundamental Theorem of Arithmetic
  - **The Not-Quite-Fundamental Theorem of Arithmetic**: Any whole number greater than 1 is either prime or it is the product of primes.
  - **The Fundamental Theorem of Arithmetic**: Any whole number greater than 1 is either prime or it is the product of primes, and its prime factorization is *unique*.
  - **Euclid's Lemma**: If a prime number is a factor of $a \; x \; b$, then it must be a factor or either $a$ or $b$.
  - $p$ is a factor of $a \; x \; b$
    - If $p$ is a factor of $a$, we are done. So suppose not.
    - $a \; x \; b \; = \; p \; x \; m$
    - $\frac{b}{m} = \frac{p}{a}$
    - Claim: $\frac{p}{a}$ is in lowest terms.
  - **Reduced Fractions Lemma:** When a fraction is reduced in lowest terms, the reduced numerator is a factor of the original numerator, and the reduced denominator is a factor of the original denominator.
    - $\frac{b}{m} \rightarrow \frac{p}{a}$
    - $p$ must be a factor of $b$, so $p$ must be a prime number.

- **Extended Euclid's Lemma**: If a prime number is a factor of a product, it must be a factor of one of the members of that product. In other words, if a prime number is a factor of $a_1 \ x \ a_2 \ x \ ... \ x \ a_n$, then it must be a factor of at least one $a_i$.
  - Euclid's extended lemma is a repeat application of Euclid's lemma.