Mathematics for Every Teacher

with Jake Tawney

Lecture 4a: Polygonal Tiling

Outline:

Which regular polygons tile the plane?

- Regular Polygons (all lines and angles inside the shape are equal):
  - Regular Triangle
  - Regular Quadrilateral
  - Regular Pentagon
  - Regular Hexagon
- **Squares** tile the plane. How do we know the angles come together?
  - Four, 90˚ angles combine to form 360˚. There are no gaps or overlaps.
- **Equilateral triangles**?
  - Why is it true that the angles come together nicely in the center?
  - If angles in a triangle add to 180˚, then angles in an equilateral triangle are 60˚
  - Six triangles come together in a way that there is no gap and no overlap.
- **Regular pentagons**?
  - What is the angle measure? Take a pentagon and divide it into three triangles.
    - Three triangles at 180˚ apiece equals 540˚ total for a pentagon.
    - 540˚ ÷ 5 = 108˚
    - When tiled, regular pentagons leave a gap of 36˚.
  - **Some regular polygons tile the plane and some do not. How do we know which regular polygons tile the plane and which do not?**
- **Regular hexagons**?
  - Triangulate the hexagon in order to determine the angle sum.
  - 180˚ x 4 = 720˚
  - Each angle in a regular hexagon is 720˚ ÷ 6 = 120˚
  - 120˚ x 3 = 360˚, there is no gap or overlap.

How do we go about testing all shapes?

Name	Number of Sides	Sum of the Angles	Measure of One Angle	How Many Shapes "Come Together"	Does It Tile?
Equilateral Triangle	3	180°	60°	6	y
Square	4	360°	90°	4	y
Regular Pentagon	5	540°	108°	~~3 4~~	n
Regular Hexagon	6	720°	120°	3	y

- As the shapes that "come together" decreases the measure of one angle increases. As the shapes that "come together" decreases to 2, the angle measure increases to 180°.
- Once you hit 3, nothing after that comes together.

Regular Polygon Tiling Theorem

- There are only three regular polygons that tile the plane: the equilateral triangle, the square, and the regular hexagon.