CLASSICALU

Teaching Math Classically
with Andrew Elizalde

Lesson 12: Constructing Mathematical Arguments

Outline:

How to construct persuasive, convincing mathematical arguments, or proofs.

- Engage students in the process of thinking about solving equations not in terms of solutions, but in terms of mathematical compositions, with the solution being the last line.

Whiteboard work (time stamp 1:15)

- Help students understand that although mathematics isn't solely a logical, axiomatic system, it has that characteristic. Students are building a consistent, coherent system when they engage in mathematics.
- We start with the notion that we have numbers, like 4, that we can do things with. When we have two numbers separated by an operator, we can achieve a value: *numerical expressions* (4 + 3). (Value is 7.)
    - What is the value of 4x + 3? The variable x makes it a *variable expression* that cannot be found in itself. The value of the expression depends on the value of the variable.
    - Is the expression equal to something? If so, then x is no longer a variable. (4:15)

$$23 = 4x + 3$$

    - The variable is now an *unknown*, and the *variable expression* becomes an *equation*.
    - Can we find the value in our heads? In this case, yes. (x = 5)
    - But what if $23.75 = 4\frac{2}{3}x + 3\frac{1}{2}$? (5:40) This one needs to show work to convey what we're doing.
    - If the students solved the earlier problem, they know the order of things, and that 3 ½ needs to be addressed first. What's the best way to show this?
- Now let's go to something simpler.

$$29 = 7x + 8$$

    - This is a balanced statement. Visualize this as a seesaw or teeter-totter which is perfectly balanced. (8:25)

- How can you keep one side from going up or down? As long as you add or subtract equally to each side, you maintain balance.

$$29 + 5 = 7x + 8 + 5$$

- But adding 5 makes it more complicated. Adding or subtracting should instead bring us closer to finding the value of x.
- So, let's go back to the original.

$$29 = 7x + 8$$

$$29 + (-8) = 7x + 8 + (-8)$$

- (10:45) The first concept is *additive inverses*. 8 has an additive inverse: (-8).
- The second concept is *inversion*. When you are solving an equation, you are "undoing" it or going in reverse order, or inversion.

$$21 = 7x + 0$$

- (11:30) (Yes, you must add the 0!)
- Inverses undo one another and leave behind elements known as *identity elements*: "+ 0" here.
- (12:00–12:30)

$$21 = 7x$$

$$\frac{1}{7} \times 21 = \frac{1}{7} \times 7x$$

$$\frac{21}{7} = \frac{7}{7}x$$

$$3 = X$$

- (12:45) Three concepts are at play with this solution:
  - maintaining balance
  - inverse operations
  - identity elements
- And carried through twice.

* Students will probably argue that this is a ridiculous amount of work to do when we already know how to solve the equation.
  - But we are establishing that mathematics is logical and not magic.
  - Things don't just "disappear" or move around for no reason.

- Andrew asks his students to show work until they feel uncomfortable with it and begin to feel the need to make the process more efficient and elegant.
  - They will want to synthesize more.
- Good arguments always take the audience into account.
  - Aristotle's definition of rhetoric is taking into account what your audience knows and using all available means to persuade them to a right action or belief.
  - Does the audience have a common knowledge that allows us not to show every step?
    - For example: "+ 0"
  - (15:45)Jumping from $29 = 7x + 8$ to $29 – 8 = 7x$ probably needs more explanation from the student to prove this is a sound step (which skips several).
  - (16:55) Examples of bad processing.
- Let the students make the case for beginning to omit the simpler sequences.
  - Does the audience have that common understanding necessary?
  - Can the student explain the process well enough to prove he or she understands it thoroughly?
    - Don't let the students skip too quickly over steps without offering evidence of their understanding of the "littler" maneuvers or concepts which are crucial for solving equations of other kinds.
  - Every mathematical move is logical.
    - If students can't give a reason for it, they shouldn't make it.

It is essential to establish early on a foundation of presenting a persuasive mathematical argument. It is mathematical rhetoric - presenting an eloquent, persuasive argument to an audience based on what they know.