Teaching Math Classically
with Andrew Elizalde

Lesson 16: Essential Elements for Teaching Math

Outline:

Introductory Lesson

Imagine you are a twelfth-century astronomer with a geocentric worldview watching stars in the night sky…

- What would be important to you?
  - patterns
  - movement
  - cycles/season
  - 0 to 360°
- How would you identify stars?
  - Pointing out stars to a friend you love with cheesy lines… (Show your enthusiasm and make it fun.)
  - By the position of a star according to its position "above" the horizon.
- How far away is the star?
  - There is no way to determine it, so we will call it a celestial unit.
  - (Remember that this is exciting stuff!)
- The distance above the horizon = sine
  - If the angle to the star is 30°, it is an equilateral triangle.
  - sin 30° = ½ celestial unit
  - We can figure with 45°, 60°, etc.
- For the "shadow length" of a further star = tangent.
- What can we extract from this?
  - If a2 + b2 = c2, then sin2 + cos2 = 12
  - $\frac{\sin \Theta}{1 \text{ c. unit}} = \frac{\text{opp}}{\text{hyp}}$ $\frac{\cos \Theta}{1 \text{ cu}} = \frac{\text{adj}}{\text{hyp}}$ $\frac{\tan \Theta}{1 \text{ cu}} = \frac{\text{opp}}{\text{adj}}$
    - This is what most textbooks will open with.
- On day 2, let's look at the stars in the rest of the sky…we have the x-y coordinate system with positives and negatives to work with.
- How about a function that inputs the angle and outputs the sine value? Four or five days later, based on the same original scenario of a medieval astronomer and early trigonometry, you could be graphing repeated cycles… From there you can delve into modern science…sound waves.
  - Is it possible to control the shape of the graph?
  - (time stamp 16:18) From a TED video, "Pamelia Kurstin plays the theremin."

        - When she is touching anything, one hand controls amplitude (height, volume).
        - The other hand controls frequency (quality of the sound).
        - She is manipulating the shape of the sine wave to make music.
- What can this science/math do for people? (17:56)
    - Dan Elsey composes and plays music with mouth/facial motion

A Framework for Classical Christian Education

- Contextualize through storytelling.
    - Tell the history of math.
    - Rehumanize math.
- Love your subject and your community out loud.
    - Show your enthusiasm (pathos)
    - Your enthusiasm can persuade your listener.
- Color your classroom with traditions and routines.
    - Put work on the board and have a conversation about the strategies used that says I value you and your work even if I'm not sure it's right.
    - a "gallery of student mathematical art"
- Teach Socratically through problem solving.
    - "How would you solve this?"
    - "How would you describe the stars?"
    - Go from concrete to pictorial to abstract to unify the concept.
- Deriving and demystifying formulas
    - applying previous knowledge to new situations or concepts
    - not because the formula is the next section in the textbook
    - where math comes from in real life and history—how humans creatively used math to solve problems
- Translate theories into application
    - theremin video
- Motivate service through doctrine and doxology
    - What does it mean to be a Christian mathematician, an educated human being?
    - stewarding good gifts God has given us
    - the beauty of the music therapy video

Unpacking the above….

- Teaching as Storytelling
    - According to David Hicks in Norms and Nobility, the difference between mytho-poetic pedagogy and an analytical pedagogy challenges us to consider the stories behind the concepts and introducing ideas through stories.
    - James K. Smith - we are "liturgical animals wooed by stories"
    - Teachers should be the best storytellers.
    - Japanese lessons, which are arguably more effective, are different in that they use long concepts, have fewer interruptions, elevate the status

of teachers, are considered "sacred," have coherence of entire lesson, and utilize storytelling.
    - Elements of a good story:
        - Rehumanizes math
            - Shows real people doing math
            - Shows what the teacher loves about math (and it's contagious)
            - A story is not always axiomatic and logical but is worth pursuing.
            - A story reveals that ideas were not prepackaged but were "discovered."
            - Slows learning down and gives space to breathe.
    - Forms rhythms of creating and resolving tension
        - In Kieran Egan's Teaching as Story Telling: a model like this sets up conflict with a resolution at the end
        - Lessons can be set up as stories with tension and resolution: "How do we make sense of the stars?"
            - The resolution of this question will span several lessons.
    - What are some good math stories we could tell?
        - Development of concept of 0
        - Approximation of π
        - Irrationality of $\sqrt{2}$
        - Development of imaginary numbers and the reaction of the math community
        - Calculus: concept of the infinitesimal, the key to understanding motion
- Loving Your Subject and Community Out Loud
    - Show your affection for the subject and let it be contagious.
    - What are teachers talking about in the hall that students hear?
        - Cardinal Newman's quote from *The Idea of a University* about students "breathing in" the teachers' "atmosphere of thought" as teachers interact with one another.
    - Demonstrate what it looks like to pursue ideas and read books
        - Andrew's advice to read *Zero: A Biography of a Dangerous Idea* led to discussions with a student on concept of zero.

- [#4 in earlier list] Teach Socratically Through Problem Solving
    - You can't teach this way every day, but it can be a part of mathematics class (not just humanities).
    - You need sufficiently complex problems students have not seen before that require previous knowledge they can piece together.

        - For instance, square roots of negative numbers. Allow class to debate their existence.
    - They will realize they don't know what to do to solve the issue.
    - Tell the story of the conflict the idea created.
    - Tell them $\sqrt{(-1)}=i$, then develop the idea for the students.
    - Help them see this is how math concepts developed.
    - An example: A Coke can in a calculus class
        - Is this the optimum design for this can?
        - How do we solve this problem? How do we judge?
        - One way might be to minimize material costs.
        - Write an equation for optimum design for 12 fluid ounces in a cylindrical object leading to optimal radius and height.
        - The result will be that the optimal height will be only slightly higher than the width of a human hand.
        - If a person would hold it in their hand, you wouldn't be able to see it.
        - The real design shows marketing appeal.
        - A purely mathematical approach does not take into account aesthetics (or marketing appeal).
        - Or maybe a Coke can is not a perfect cylinder.
    - Another example: If you add an even # to an even #, do you always get an even #?
        - Introduce the idea of certainty.
        - Is it worthwhile to engage in a discussion of this with a younger mind?
    - Introduce this way of making proofs and justifications and rhetoric very young and across the curriculum. Rhetoric can be part of elementary school.
- [#5 in earlier list] Derive and Demystify Formulas
    - Engage students' senses as often as possible.
    - Move from concrete to pictorial to abstract.
    - Work with physicalities and manipulatives
        - What does 100 feel like? What is its weight?
        - How does 10 feel? How does 1 feel?
    - For example, allow the student who least understands subtraction and regrouping to physically subtract from groupings or units of higher value.
        - It is "decomposing" or "regrouping" (not "borrowing" or "carrying over").
        - The student will remember the physical act.
    - Physicality and manipulatives aren't just for little kids.
        - For example, geometry aerobics with motions (time stamp 42:40).

        - The more senses that are engaged, the more memorable—neural plasticity.
        - Get silly with it if necessary.
- [#3 in earlier list] Color Classrooms with Traditions and Routines
    - James K. Smith: Teaching as a formative practice.
        - It tunes affections and expresses value.
    - Teach the importance of the practice and process.
    - Mathematics is about compositions rather than strictly solutions.
        - The communication of the work is important and must be shown.
        - It invites others in.
        - It's not just about exposing errors.
        - Even mistakes are valuable.
        - Creativity or some other aspect can still be valuable to celebrate student's engagement in the process, even if they are wrong.
    - Example of dividing the class into groups to compete to complete a complex problem with talking only at halftime.
        - Each student must work based on previous student's (hopefully) clear arguments.
    - Expressing value in your classroom through daily routines is habitus.
        - James K. Smith references this concept based on Etienne Wenger.
        - It is forming a concept of what is valuable and good.
        - orientation and inclination that students absorb in their day-to-day practice
- Translating Theories into Application
    - Show students what practical things can be done with ideas
    - (These practical applications were shown in the videos earlier.)
- Motivate Service through Doctrine and Doxology
    - Is our subject strictly utility—we need math for making brownies, etc.?
        - Students won't buy this.
    - Even in Plato's *Republic* (book 7), mathematics is on a trajectory toward natural philosophy and ultimately theology.
        - It informs how you think about yourself, God, and the world.
        - The ancients assumed that arithmetic, geometry, and music would lead a person to think about God.
    - As Christian educators, why are we doing this? What is our biblical telos?
        - Steward the intellect by cultivating it so that we can be useful to God for His calling upon us.
            - God will use what I learn, so I need to learn well.
        - Not grades (necessarily) but formation of character
        - Ask more gifted students to read an extra book and discuss it.
            - Working together and investing in the student.
        - Service through doctrine and doxology

- Romans is full of theology and doctrine.
- Romans 11 ends with a doxology/worship.
- Romans 12 prompts us to service.
- Historical example:
    - In the appendix of Isaac Newton's *Principia*, he writes how in awe he is about the stars and the planets—they must be held together by a universal being. And it is not sufficient to call Him a universal being but Lord, and we must worship him.
- Stand in wonder and awe yourself, and bring historical figures' awe to students' attention.
    - *Mathematics: Is God Silent?* by James Nickel: the first half is a compilation of Christian mathematicians' words of marvel at God

- Help students grow in wisdom and understand the limits of what they're doing.
    - Alister McGrath *in The Science of God* compares scientific enterprise to the creation of systematic theology.
        - Scripture wasn't delivered systematically, so systematic theologians need ontological humility.
    - There are paradoxes, flaws, mysteries, and problems of math and scientific theory, but mathematics is a language that explains creation so well. It's a blessing. (See Thomas Koonz, *The Structure of Scientific Revolution*.)
    - Students should not have a posture before their subject area of mastery or dominion but one of gratitude and service.
    - Knowledge of math should help students navigate apologetics and origins.
    - Math is a way to serve and love our neighbors—make, go, and tell the Good News.
    - Realize that the universe is well ordered. The beauty and order of math and science can lead to faith.

Which of all these things are how I teach?

Which are things I want to learn more about?