Teaching Math Classically
with Bill Carey

Lesson 17: Mathematics as a Humanities Subject

Outline:

(1:11) **Q: Is mathematics a humanities subject?**

- We often have favorite songs, books, movies, artworks…but favorite mathematical equations?
- We usually don't think of math as "art," but it is—it's a way of communicating beautiful things about the world to each other.
- Math sometimes seems very "other" and "scary" as a subject. The patterns seem so different. In fact, it can be like the Greek myth of Sisyphus. His punishment from the gods was to roll a boulder up a hill. When it slipped he had to do it all over again throughout eternity.
- Math can feel like we are pushing an eternal boulder. We have a set of exercises—but there are more that are harder. And then after you finish those, there are still more problems.
  - What are you being punished for?
  - I could do math, but why would I *want* to?
  - When you make art, you have a work at the end, but when you do math what do you have at the end of it?
- But math *can* look like an English class. We can think about math like we think about the humanities.
  - If we look at math as a humanity, it gives a reason why people would do it *on purpose*.
  - Utilitarian reasons aren't very compelling, but when we look at math as art, we have a different motivation to do it.

(8:00) **Q: Do adults in the real world view math as art?**

- It turns out that professional mathematicians write *papers* to each other. How do mathematicians evaluate each other? They judge its truth of course, but mostly they judge its elegance or beauty. If it's clever or funny, it's approved.
- Example: Are there more fractions or are there more integers? How do you argue your position?
  - There are infinity fractions between 0-1. *But* there are infinite integers also.
  - If you can make up a rule that connects every integer to one fraction, then you can prove there are the same number of both.

And mathematicians have made this rule to prove it. (See Calkin and Wilf, University of Pennsylvania.)

- Mathematicians consider the papers *art.* But what are some other math professions?
    - Bill makes maps/digital cartography. Computer programs translate and identify features of satellite maps. Teaching computers to do all this is math.
    - He uses one math problem every *two months.*
    - The answers are not in the back of the book.
    - How does he check his math work? Someone else goes over it and finds weaknesses. Bill tries to justify his process and the other person objects until together they figure out how to do it right. After a couple of weeks Bill has a rough draft composed of computer code/equation/English paragraphs.
    - Once again, a larger group will analyze it and criticize its processes until they reach a solution they can use to make maps.
    - This doesn't have much resemblance to how math is often taught in the classroom in high school.
    - "Grown up" math is collaborative. It may not be correct at first. The answer is a paper or program rather than, for example, "3.
- Taking a detour…

Note this series of numbers:

	If you add them together, you get:
1	1
1 3	4
1 3 5	9
1 3 5 7	16

These numbers (in the second column) are all squares! Does this pattern go on forever?

- So, using the last line as an example, if $x = 4$, then the second column would be x^2. Then the previous square in the second column would be $(x-1)^2 \rightarrow x^2 - (x-1)^2$
- If you subtract 9 from 16, you get 5, which is the last number on the list in the first column for $(x-1)^2$…

$$x^2 - (x-1)^2$$

$$x^2 - [(x-1)(x-1)]$$

$$x^2 - (x^2 - x - x + 1)$$

$$x^2 - x^2 + 2x - 1$$

$$2x - 1$$

- *(23:16)* If x is the square root of each line's sum, the expression above gives you the last odd number for the line. And if you subtract the line's sum from the sum of the line after it, you get the last number in that line.
    - This convinced Bill that the pattern goes on forever.

(24:17) Hough Transform

$$(1 + 2)^2$$

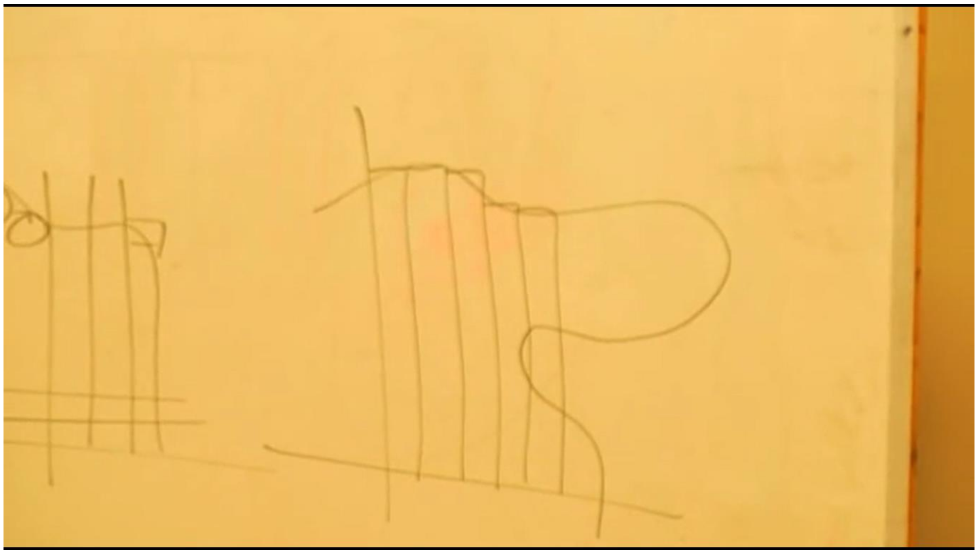
$$3^2$$

$$9 \rightarrow (x + y)^2 \rightarrow (x + y)^z$$

- It becomes more general.
- For Bill's job, when you solve a math problem, you want to solve it for several different things at once.
    - In other words, you don't want the map program equation to solve for one river, but a lot of different rivers.
    - You will have multiple input points for multiple outputs.
- You are often wrong in this process.
    - You are doing drafts that you refine in process with others.
    - Bill does it for seven or eight times on average, but in one case (identifying power lines), it took him about eighty drafts/tries.
    - It is a process that needs input from peers and multiple revisions.

(27:25) **Q: What is your approach to teaching math that involves problems, exercises, and essays? Why do you do that?**

- **A:** Students often ask "Who cares?" when they do math. So—although there are some math things we just need to absolutely know—Bill began to give students problems that they couldn't solve.
- Students came up with solutions based on previous knowledge…finding the area of circles, squares inside an oddly shaped figure to determine its area.
- So then they came up with more solutions—stacks of rectangles on a graph. (This is actually how mathematicians have agreed to solve this kind of problem.) The class spent time together working out a formula.



- Bill could have just given them the formula, but the point was that students now know *why* it's true and how the formula fits with the problem they're trying to solve.
    - They know what the formula is *for* and what its context is.
- The students practice with the formula for a number of exercises but also write group essays (about five to seven pages). For example, "Which size cup at Starbuck's is the best deal? Justify your answer."
    - What does "best deal" mean?
    - What drinks qualify?
    - Some students spent the first part of their essay explaining why the question was bogus but then choosing a standard for "best deal" and which drink and working from that.

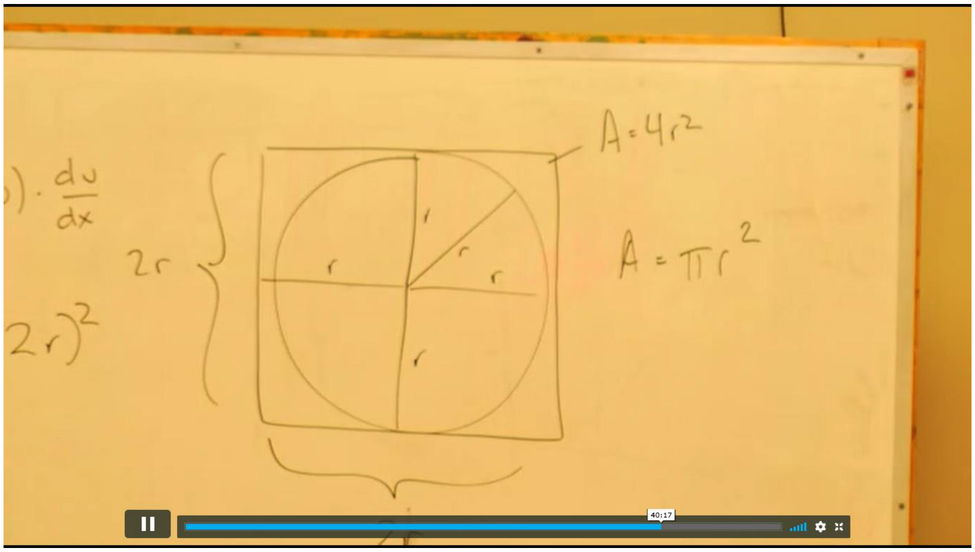
(34:30) **Q: What is your favorite math equation and why is it beautiful?**

- **A:** The chain rule in calculus is a sublime joy.

$$\frac{d}{dx} f(u) = f'(u) * \frac{du}{dx}$$

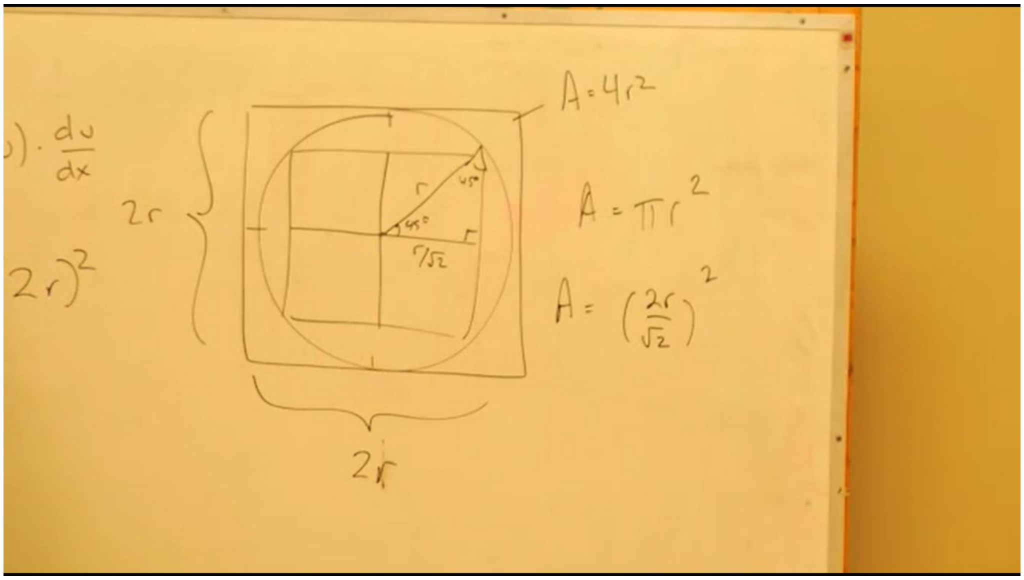
- Bill meets with students for dinner and math (it's a math club). It's a lot more fun just to play with an idea rather than doing a lot of problems fast under pressure. It's relaxing like improvising with a musical instrument.
- His school had a pi contest to see who could memorize most of pi's value. The reward was, of course, pie.
- How do we know that $\pi = 3.14\ldots$? (Besides that it's in the books.) How did Archimedes know?
    - He used the Method of Exhaustion.

- *(38:40)* Can we say anything about the area of a circle? We know it's πr^2.
- *(30:35)* Archimedes drew a square around the circle. Its area is $(2r)^2$ or $4r^2$.
  - Since the circle is smaller than the square, we know π is smaller than 4.



- If we draw another square *inside* the circle, the diagonal is r. We can figure out the area of the square with previous knowledge.

$$(42{:}50)\quad A = \left(\frac{2r}{\sqrt{2}}\right)^2$$



- Since the area of the square is smaller than that of the circle, we know that π is larger than this area.

- So, what if we draw a hexagon inside the circle? Its area would be closer to the circle's area. We could draw an octagon—that would be even closer. Or we could draw a decagon or a million-sided polygon, and each would get us closer to the value of π.
- It took work to get to π—it didn't come from a blinding flash.

(44:45) Students often get numb to the idea that established values or formulas come from somewhere.

- But figuring this out is what makes math fun.
- What's not fun is just doing the work by rote. It's pushing Sisyphus's boulder up the hill. It's like taking art and writing out the definitions in terms of color and composition and then in a more advanced class, you learn all the names of musical notes. Next in musical theory you memorize chords and compose, but *you never listen to music.* The point of the whole thing is to make and listen to beautiful music.
    - That's what we often miss in math—the beauty and aesthetics of it.
- Bill didn't love math until his third year of teaching. He was good at it but he saw it as a "cookbook." He liked it but he didn't love it until he realized there was more to it, more behind the surface.

(48:40) **Q: How do you have fun when the problem takes two months to finish?**

- **A:** Math is made up of lots of little math problems. Solving the parts that will make up the larger whole is satisfying.
    - You have a vision you are working toward.
    - You often have the freedom to explore. If Bill gets stuck or frustrated, he will work on something else for a while. But then he will get an idea to try something with the original problem.
    - Like anything creative, being pressured or forced to be instantly creative with math doesn't work.