Teaching Math Classically
with Andrew Elizalde

Lesson 9: Teaching Math with Socratic Dialogue – Part 2

Outline:
To implement Socratic dialogue in the classroom, begin lessons with a sufficiently complex problem that would lend itself to the Socratic method.

According to Richard Riesen's The Academic Imperative: A Reassessment of Christian Education's Priorities, the virtues of the Socratic method are the following:

- While the Socratic method doesn't create or demand leisure, it encourages that kind of thinking or thoughtfulness for which leisure is the most congenial circumstance.
- It inculcates the skills and tempers required for civil discourse. In a society in which civil discourse is conspicuously rare, giving young people an opportunity to participate in it can only be worthwhile.
- Giving students practice in civil discourse offers opportunities for the exercise of Christian charity.

The Socratic method is character forming, teaching students how to speak to and critique one another charitably and how to engage in conversation.

The Socratic method also gives room for intuition and imagination.

- Mathematics is not always step-by-step or logical argument.
- The discussion nature allows flexibility to explore ideas and gives permission to go down wrong paths before retreating back to new ones.
- The Socratic method presents a more real picture of what the actual process of doing mathematics looks like.
    - Morris Kline, in *Why Johnny Can't Add*, quotes mathematician Henri Lebesgue: "No discovery has been made in mathematics—or anywhere else, for that matter—by an effort of purely deductive logic. It results from the work of creative imagination, which builds what seems to be truth, guided sometimes by analogies, even sometimes by an aesthetic ideal, but which does not hold at all on a completely solid logical basis. Once a discovery is made, logic intervenes to act as a control. It is logic that does ultimately decide whether the discovery is really true or illusory. Its role, therefore, though considerable, is secondary. Thus, the concentration on a purely deductive approach omits the vital work. It destroys the life and spirit of mathematics."

The Socratic method dovetails with the idea of teaching through problem solving.

- It presents problems on the front end for students to work through, rather than always having students merely rehearse what the teacher modeled in the classroom.
- It allows students to bring out their own ideas for solving, or engages them in dialogue to test their ideas' value.

<u>Mathematical Concept Problem-Solving That Lends Itself to Socratic Dialogue</u>

- Graphing rational functions for the first time--assuming students have never seen this function before (make sure they have not peeked in the textbook beforehand and the books stays closed):
- Consider this function and allow students to fill out the table of values, likely something like this (*Graphing of these values at time stamp 6:40*):

X	Y
-1	-1
1	1
2	½

$$y = \frac{1}{x}$$

  - **Ask** students: "How can we determine the shape of the graph?"
    - Let the students come up with ideas for what the graph will look like, then justify their answers.
    - Allow them to make mistakes.
    - Likely they will think in terms of straight lines and connecting the points between the positive and negative sectors.
  - **Ask:** "How do we know what the graph will look like? How can we test it? Is the table of values complete? Could the values of x between -1 and 1 be tested? What about 0?"
    - Let students add to the table of values:

X	Y
-1	-1
½	2
3	1/3
0	

  - **Ask:** "Does the graph exist between 1 and -1? Why can't we graph at 0? Why limit to integers?" (Time stamp for graphing: 10:15)
    - Experiment with various values. (Time stamp for graphing: 11:05)
    - Lead students to the belief that the graph has unusual behavior around the origin.
  - **Ask:** "Can we do the same method with negative values?"
    - Add to the table of values with negative values.
    - Challenge the students on whether the two sections can be connected over the 0 value.

    - After they work through it, tell them that this conversation has been had before by mathematicians, and the behavior of the graph is called a vertical asymptote.
  - **Ask** students whether they think the graph goes into positive and negative infinity. Ask: "Is this graph an accurate graph of the function of y=1/x ? Have we vetted that enough?"
    - Require students to go back through previous arguments and sequence them. Did they consider what is happening for larger values of x? Graph those values. (Time stamp: 16:44)
  - **Ask:** "Will the graph cross over the x axis? Will it ever settle onto it or touch it? How do we demonstrate this?"
    - Let them try the values of 100 or 900, for example. With 900, they are now using their imaginations.
    - Let them justify their ideas even if they are correct. Keep challenging their conclusions.
  - After they have justified their answers, present the idea of infinitely smaller fractions, as with the example of one pizza sliced into increasingly smaller pieces until the serving is mere "pizza dust."
  - (You can spend a whole class period guiding students through the discussion of the thought behind the graph of this function.)
- Now consider: $y = \frac{1}{x+3}$
  - Students will likely conclude that the graph behaves strangely at -3, but repeats a similar pattern to the previous function. (23:20)
  - But what if the function is: $y = \frac{x-5}{x^2-2x-15}$?
    - Students again will recognize the graphing pattern.
  - How about if the denominator is factorable into to $y = \frac{x-5}{(x-5)(x+3)}$, and then we get to $y = \frac{1}{x+3}$?
  - **Ask:** "When we eliminate (x–5), have we eliminated something graphically necessary from $y = \frac{x-5}{x^2-2x-15}$? How can we determine that? Should we graph various values for x?" (Steer them away from 5 for the time being.) (27:50)
    - As they graph more values, students will begin to see that the graph looks like the one for: $y = \frac{1}{x+3}$.
  - Keep questioning what students have left out. **Ask:** "Have we plotted every value?" (Of course not.) "But what haven't we graphed? How about 5?"
    - With $y = \frac{1}{x+3}$, when x = 5, y = 1/8.
    - But if students work out x = 5 in the original function $\left(y = \frac{x-5}{x^2-2x-15}\right)$ they will come to $y = \frac{0}{0}$.
  - **Ask:** "Can we divide 0 by 0?" (No.) "That's odd. The graph doesn't seem to fit the pattern. Let's try x = 4.99." Then $y = \frac{1}{7.999}$.

        - We can get closer to 5, but we can't define it. (34:10)
        - The graph exists, except at 5…
    - "What is the name for this strange behavior? Historically mathematicians have called this a *removable discontinuity*." You will have an opportunity to present some new vocabulary.
    - **Conclusion:** Rational functions present a rich context for good Socratic dialogue, drawing out questions and justifications for students' propositions.

Socratic dialogue teaches students to be resilient, to be reflective, to vet their ideas over and over, and to be willing to be critiqued.

- It demonstrates what it looks like to
    - come up with ideas
    - test and question ideas
    - Work in community.
- Students might never graph a rational function after high school or college, but they will almost certainly have to brainstorm and test ideas in concert with others.

Challenge: Find one part of your curriculum in which you can present a new, complex problem. Put away the textbook and allow students to work through it with Socratic dialogue rather than laying out the answer before them.